Theory for quantum state of photon pairs generated from spontaneous parametric down conversion nonlinear process

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Abstract

We present a theory for the quantum state of photon pairs generated from spontaneous parametric down conversion nonlinear process in which the influence of the final sizes of nonlinear optical crystals on eigen optical modes is explicitly taken into consideration. We find that these photon pairs are not in entangled quantum states. Polarization correlations between the signal beam and the idler beam are explained. We also show that the two photons generated from SPDC are not spatially separated, therefore the polarization correlation between the signal and idler beams is not an evidence for quantum non-locality.

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I. INTRODUCTION

The entangled quantum state is the core of Eistein-Podolsky-Rosen (EPR) paradox [1], and forms a base for many possible application of quantum information. Photon pairs generated from spontaneous parametric down conversion (SPDC) nonlinear process are generally considered as entangled photon pairs. They are widely used in experiments that require entangled photon pairs as light sources. By applying a quantum transition theory, Shi and co-workers showed that photon pairs generated from SPDC nonlinear process are quantum entangled [2]. But however, in their theory, the finite sizes of nonlinear optical crystals were not taken into consideration. According to their simplification, the eigen modes of optical field are plan waves, even in the presence of optical crystals. But as pointed out by N.Bloembergen [3], the correct eigen modes of optical field, in this case, are linear combinations of plan waves determined by boundary conditions at the surfaces of optical crystals. Because the quantum state of photons generated from SPDC nonlinear process depend explicitly on the eigen optical modes, the correctness of Shi and co-workers' conclusion on the quantum state of photon pairs generated from SPDC is a question.

In this paper, we present a quantum theory for the quantum state of photon pairs generated from SPDC, in which the effect of the finite size of optical crystals is explicitly taken into consideration. We find that these photon pairs are not in entangled quantum states, and the correlation between the polarizations of photons is not an evidence for quantum non-locality.

A hamiltonian that describes SPDC process is introduced in Sec.II, and the eigen optical modes are analyzed in Sec.III. An explicit expression for the quantum state of photon pairs generated from SPDC is established in Sec.IV. This expression for the quantum state is used in Sec.V for analyzing the polarization correlation between the signal and idler beams.

II. THE HAMILTONIAN FOR SPONTANEOUS PARAMETRIC DOWN CON-VERSION PROCESS

Let's consider an optical crystal with the second order optical nonlinearity. The effective Hamiltonian for the optical parametric process is given by [2]:

$$H_1 = \varepsilon_0 \int_V \sum_{i,j,h=1}^3 \chi_{lmn} E_{pi}^{(+)} E_{sj}^{(-)} E_{sh}^{(-)} d^3 \vec{r} + h.c.$$
 (1)

where ε_0 is the dielectric permittivity of the vacuum, V is the volume of the optical crystal, χ is the second order nonlinear electric susceptibility tensor, $\vec{E}_p^{(+)}$ is the positive frequency part of the pump optical field, $\vec{E}_s^{(-)}$ is the negative frequency part of the generated optical field, and h.c. means the Hermite conjugate.

We expand optical fields $\vec{E}_p^{(+)}$ and $\vec{E}_s^{(-)}$ into linear combinations of eigen optical modes:

$$\vec{E}_p^{(+)} = \sum_{\omega_p, l} b_l(\omega_p) \vec{u}_l(\omega_p, \vec{r}) \exp(-i\omega_p t) \text{ and } \vec{E}_s^{(-)} = \sum_{\omega_s, l} b_l^{\dagger}(\omega_s) \vec{u}_l^*(\omega_s, \vec{r}) \exp(i\omega_s t),$$
 (2)

where $b_l(\omega)$ is the annihilation operator for a photon with the circle frequency ω in the eigen mode $\vec{u}_l(\omega, \vec{r})$. The eigen mode $\vec{u}_l(\omega, \vec{r})$ satisfied the following orthonormality conditions:

$$\int \sum_{j,k=1}^{3} \varepsilon_{jk} u_{lj}(\omega, \vec{r}) u_{mk}^*(\omega, \vec{r}) d^3 \vec{r} = \frac{1}{2} \hbar \omega \delta_{lm}, \tag{3}$$

and

$$\int \sum_{j,k=1}^{3} \varepsilon_{jh} u_{lj}(\omega_1, \vec{r}) u_{mh}^*(\omega_2, \vec{r}) d^3 \vec{r} = 0, \quad \text{if} \quad \omega_1 \neq \omega_2.$$
 (4)

By applying relations (2),(3) and (4), we obtain the following expression for the total Hamiltonian:

$$H = \sum_{\omega} \sum_{l} \hbar \omega b_{l}^{\dagger}(\omega) b_{l}(\omega) + H_{1}, \tag{5}$$

with

$$H_1 = \sum_{l,m,m'} M(l,m,m') b_l(\omega_p) b_m^{\dagger}(\omega_s) b_{m'}^{\dagger}(\omega_s') \exp[-i(\omega_p - \omega_s - \omega_s')t] + h.c., \tag{6}$$

where

$$M(l, m, m') = 2\varepsilon_0 \int_V \sum_{i, j, h=1}^3 \chi_{ijh} u_{li}(\omega_p, \vec{r}) u_{mj}^*(\omega_s, \vec{r}) u_{m'h}^*(\omega_s', \vec{r}) d^3 \vec{r}.$$
 (7)

According to the quantum transition theory, SPDC may occurs only $\omega_p = \omega_s + \omega_s'$ and $M(l, m, m') \neq 0$. To calculate the matrix element M(l, m, m'), one needs explicit expression of eigen optical mods. One may observe that due to the presence of a optical crystal of a finite size, plan waves are no longer eigen modes of the optical field.

III. EIGEN OPTICAL MODES

We consider eigen optical modes in presence of a $L_x \times L_y \times L_z$ uniaxial optical crystal, with the optical axis oriented in the direction $\vec{e}_a = (\sin \theta, 0, \cos \theta)$.

According the expression (7), one needs only the expression for eigen optical modes in the optical crystal. Within the optical crystal, the eigen optical modes are linear combinations of plan waves that are reflected into each other at the crystal's surface. One may observe that the constant of normalization is not important in determining the quantum state of photon pairs generated from SPDC.

In real applications, the optical axis is so oriented that the integration (7) is significantly different from zero only for optical modes containing plan waves component with $|k_z| \gg |k_x|, |k_y|$. These plan waves are totally reflected at surfaces with $x = \pm L_x/2$ and at surfaces with $y = \pm L_y/2$, but weakly reflected at surfaces with $z = \pm L_z/2$. We will neglect reflections at $z = \pm L_z/2$. We also neglect the coupling between o-beams and e-beams due to reflections at $z = \pm L_x/2$ and $z = \pm L_y/2$, because the principal plans are either nearly parallel or nearly perpendicular to the plans of incidence in these cases.

By using these approximations, we may separate eigen optical modes into o-modes and e-modes. We have the following expression for the m-th o-modes, within the crystal:

$$\vec{u}_o(k_{mx}, k_{my}; \vec{r}) = N_o(m) \sum_{l=1}^4 \vec{e}_o(\vec{k}_{ml}) \exp(i\vec{k}_{ml} \cdot \vec{r} + i\phi_l), \tag{8}$$

where ϕ_l are phase factors determined by boundary conditions at $x = \pm L_x/2$ and $y = \pm L_y/2$, $N_o(m)$ a normalization constant, $k_{mx} > 0$, $k_{my} > 0$,

$$\vec{k}_{m1} = (k_{mx}, k_{my}, k_{oz}), \ \vec{k}_{m2} = (k_{mx}, -k_{my}, k_{oz}), \ \vec{k}_{m3} = (-k_{mx}, k_{my}, k_{oz}), \ \vec{k}_{m4} = (-k_{mx}, -k_{my}, k_{oz}), (9)$$

with $k_{oz} = \sqrt{n_o^2 k_0^2 - k_{mx}^2 - k_{my}^2}$, where $k_0 = \omega/c$ and n_o is the refractive index of o-beams, and $\vec{e_o}(\vec{k}_{ml})$ a vector of unity that is perpendicular to \vec{k}_{ml} and $\vec{e_a}$. Similarly, for the m-th

e-modes, we have

$$\vec{u}_e(k_{mx}, k_{my}; \vec{r}) = N_e(m) \sum_{l=1}^4 \vec{e}_e(\vec{k}_{ml}) \exp(i\vec{k}_{ml} \cdot \vec{r} + i\phi_l), \tag{10}$$

where $N_e(m)$ is a normalization constant,

$$\vec{k}_{m1} = (k_{mx}, k_{my}, k_{ez}), \ \vec{k}_{m2} = (k_{mx}, -k_{my}, k_{ez}), \ \vec{k}_{m3} = (-k'_{ex}, k_{my}, k_{ez}), \ \vec{k}_{m4} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m4} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m5} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m6} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m6} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m7} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m8} = (-k'_{ex}, -k_{my}, k_{ez}), \ \vec{k}_{m9} = (-k'_{ex}, -k_{my$$

with k_{ez} is the z-component of the wave vector of an e-beam with x and y-component of the wave vector given by k_{mx} and k_{my} , and $-k'_{ex}$ the x-component of the wave vector of an e-beam with z and y-component of the wave vector given by k_{ez} and k_{my} ($k_{mx} > 0, k'_{ex} > 0, k_{my} > 0$). $\vec{e}_e(\vec{k}_{ml})$ is a vector of unity that defines the polarization of an e-beam with the wave vector \vec{k}_{ml} . Due to the symmetry of the crystal, eigen optical modes can be classified into even modes with

$$\exp(i\phi_1) = \exp(i\phi_2)$$
 and $\exp(i\phi_3) = \exp(i\phi_4)$ (12)

and odd modes with

$$\exp(i\phi_1) = -\exp(i\phi_2) \text{ and } \exp(i\phi_3) = -\exp(i\phi_4)$$
(13)

In calculations for quantum correlation between two photons generated from SPDC, we need also the expression for optical field outside the crystal. For optical modes with $k_x = 0$, we have,

$$\vec{u}_{o}(0, k_{y}; \vec{r}) \propto \left(\vec{e}_{y} + \vec{e}_{x} \frac{k_{y}}{n_{o}k_{0}} \cot \theta - \vec{e}_{z} \frac{k_{y}}{k_{0}}\right) \exp(ik_{z}z + ik_{y}y)$$

$$+ \left(\vec{e}_{y} - \vec{e}_{x} \frac{k_{y}}{n_{o}k_{0}} \cot \theta + \vec{e}_{z} \frac{k_{y}}{k_{0}}\right) \exp(ik_{z}z - ik_{y}y + i\phi),$$

$$(14)$$

and

$$\vec{u}_e(0, k_y; \vec{r}) \propto \left(\vec{e}_x - \vec{e}_y \frac{k_y}{n_o k_0} \cot \theta\right) \exp(ik_z z + ik_y y) + \left(\vec{e}_x + \vec{e}_y \frac{k_y}{n_o k_0} \cot \theta\right) \exp(ik_z z - ik_y y + i\phi),$$
(15)

with $k_z = \sqrt{k_0^2 - k_y^2}$.

IV. THE QUANTUM STATE OF PHOTON PAIRS

Suppose that the pump photon is in the quantum state $|\psi_p\rangle$, and the generated photon pair is in the quantum state $|\psi\rangle$. According to quantum transition theory, we have the following relation between $|\psi\rangle$ and $|\psi_p\rangle$:

$$|\psi\rangle \propto \sum_{l,m,m'} \sum_{\omega_s} M(l,m,m') b_l(\omega_p) b_m^{\dagger}(\omega_s) b_{m'}^{\dagger}(\omega_p - \omega_s) |\psi_p\rangle.$$
 (16)

In the case that pump beam is a plan wave propagating in the direction \vec{e}_z , we have

$$\sum_{l} b_{l}(\omega_{p}) \vec{u}_{l}(\omega_{p}, \vec{r}) |\psi_{p}\rangle = \vec{e}_{p} \exp(ik_{p}z) |0\rangle$$
(17)

According to expressions (7) and (16), we obtain then

$$|\psi\rangle \propto \sum_{m,m'} \sum_{\omega_s} \int_{V} \sum_{i,j,h=1}^{3} \chi_{ijh} e_{pi} u_{mj}^*(\omega_s, \vec{r}) u_{m'h}^*(\omega_s', \vec{r}) \exp(ik_p z) d^3 \vec{r}$$

$$b_m^{\dagger}(\omega_s) b_{m'}^{\dagger}(\omega_p - \omega_s) |0\rangle. \tag{18}$$

There are two types of phase match conditions [4]: type-I, where the integral in (18) is significantly different from zero only if both modes are either o-modes or e-modes, and type-II, where one of these two modes is an o-mode, and another is an e-mode. We will consider only the type-II phase match in this paper. The case of type-I phase match can be treated in a similar way.

In the case of type-II phase match, the expression (18) can be written as

$$|\psi\rangle \propto \sum_{m_{o},m_{e}} \sum_{\omega_{s}} \sum_{l,l'=1}^{4} \int_{V} \sum_{i,j,h=1}^{3} \chi_{ijh} e_{pi} e_{oj}(\vec{k}_{m_{o}l}) e_{eh}(\vec{k}_{m_{e}l'}) \\ \exp[i(k_{p}z - \vec{k}_{m_{o}l} \cdot \vec{r} - \vec{k}_{m_{e}l'} \cdot \vec{r} - \phi_{l} - \phi_{l'})] d^{3}\vec{r} \ b_{m_{o}}^{\dagger}(\omega_{s}) b_{m_{e}}^{\dagger}(\omega_{p} - \omega_{s}) |0\rangle.$$
 (19)

If the optical crystal is large enough, then the integral in the above expression is significantly different from zero only for mode pairs satisfying the condition

$$\vec{k}_{m_01} + \vec{k}_{m_e4} = k_p \vec{e}_z \text{ or } \vec{k}_{m_02} + \vec{k}_{m_e3} = k_p \vec{e}_z.$$
 (20)

In these cases, we have

$$|\psi\rangle \propto \sum_{m_o} \sum_{\omega_s} c(m_o, m_e) b_{m_o}^{\dagger}(\omega_s) b_{m_e}^{\dagger}(\omega_p - \omega_s) |0\rangle,$$
 (21)

where

$$c(m_o, m_e) = \begin{cases} 1 & \text{if both of } m_o \text{ and } m_e \text{ are even or odd} \\ 0 & \text{other cases} \end{cases} , \tag{22}$$

and m_e is determined by the condition (20).

According to the expressions (14) and (15), one may observe that the optical fields of an o-mode and an e-mode may perfectly overlap outside the optical crystal if $k_{m_ox} = k_{m_ex} = 0$, $k_{m_oy} = k_{m_ey} = k_{yd}$ and $\omega_p = 2\omega_s$. If Eq. (20) is satisfied by these modes, then this kind of photon pairs can be generated by SPDC. By using the expression (21), we find the quantum state for such a photon pair as

$$|\psi\rangle = b_o^{\dagger}(0, k_{yd})b_e^{\dagger}(0, k_{yd})|0\rangle. \tag{23}$$

We used (k_{mx}, k_{my}) as the label of the mode in the above expression.

The optical field of these photon pairs is an overlap of $b_o(0, k_{yd})\vec{u}_o(0, k_{yd}; \vec{r})$ and $b_e(0, k_{yd})\vec{u}_e(0, k_{yd}; \vec{r})$. We have, outside the optical crystal

$$\vec{E}^{(+)}(\vec{r}) \propto \vec{\mathcal{E}}_s(\vec{r}) + \vec{\mathcal{E}}_i(\vec{r}), \tag{24}$$

where

$$\vec{\mathcal{E}}_s(\vec{r}) = \left[b_o(0, k_{yd}) \left(\vec{e}_{ys} + \vec{e}_x \frac{k_{yd} \cot \theta}{n_o k_0} \right) + b_e(0, k_{yd}) \left(\vec{e}_x - \vec{e}_{ys} \frac{k_{yd} \cot \theta}{n_o k_0} \right) \right] \exp(i\vec{k}_s \cdot \vec{r}) \quad (25)$$

and

$$\vec{\mathcal{E}}_i(\vec{r}) = \left[b_o(0, k_{yd}) \left(\vec{e}_{yi} - \vec{e}_x \frac{k_{yd} \cot \theta}{n_o k_0} \right) + b_e(0, k_{yd}) \left(\vec{e}_x + \vec{e}_{yi} \frac{k_{yd} \cot \theta}{n_o k_0} \right) \right] \exp(i\vec{k}_i \cdot \vec{r} + i\phi), \quad (26)$$

with
$$\vec{e}_{ys} = \vec{e}_y - \vec{e}_z k_{yd}/k_0$$
, $\vec{e}_{yi} = \vec{e}_y + \vec{e}_z k_{yd}/k_0$, $\vec{k}_s = (0, k_{yd}, \sqrt{k_0^2 - k_{yd}^2})$ and $\vec{k}_i = (0, -k_{yd}, \sqrt{k_0^2 - k_{yd}^2})$.

In the most of real experiments, pump beams are Gaussian beams. We have in these cases

$$\sum_{l} b_{l}(\omega_{p}) \vec{u}_{l}(\omega_{p}, \vec{r}) |\psi_{p}\rangle = \vec{e}_{p} \int \exp\left(-\frac{k_{x}^{2} + k_{y}^{2}}{2\sigma^{2}}\right) \exp\left[i(\sqrt{k_{p}^{2} - k_{x}^{2} - k_{y}^{2}}z + k_{x}x + k_{y}y)\right] dk_{x} dk_{y} |0\rangle\langle 27\rangle$$

and the quantum state for photon pairs with $\omega_s = 1/2\omega_p$ is given by

$$|\psi\rangle \propto \sum_{m_{o},m_{e}} \left[\exp\left(-\frac{(k_{m_{o}1x} + k_{m_{e}4x})^{2} + (k_{m_{o}1y} + k_{m_{e}4y})^{2}}{2\sigma^{2}}\right) + \exp\left(-\frac{(k_{m_{o}2x} + k_{m_{e}3x})^{2} + (k_{m_{o}2y} + k_{m_{e}3y})^{2}}{2\sigma^{2}}\right) \right]$$

$$\delta(k_{p} - k_{m_{o}1z} - k_{m_{e}1z})b_{o}^{\dagger}(m_{o})b_{e}^{\dagger}(m_{e})|0\rangle.$$
(28)

 k_{mx} and k_{my} have discretely values, with $\Delta k_x \sim 1/L_x$ and $\Delta k_y \sim 1/L_y$. Because L_x and L_y are much greater than the wavelength, so we may replace the summation over m by integration over k_x and k_y . Let

$$\pm k_{m_o x} \to k_{ox}, \ \pm k_{m_o y} \to k_{oy} + k_{yd}, \ k_{m_e x}, -k'_{ex} \to k_{ex}, \ \pm k_{m_e y} \to k_{ey} + k_{yd},$$
 (29)

we may rewrite expression (28) as

$$|\psi\rangle \propto \int dk_{ox} \int dk_{ex} \int dk_{oy} \int dk_{ey} \exp\left(-\frac{(k_{ox} - k_{ex})^2 + (k_{oy} - k_{ey})^2}{2\sigma^2}\right)$$
$$\delta(a_{ox}k_{ox} + a_{ex}k_{ex} + a_{oy}k_{oy} + a_{ey}k_{ey})b_o^{\dagger}(k_{ox}, k_{oy})b_e^{\dagger}(k_{ex}, k_{ey})|0\rangle, \tag{30}$$

where

$$a_{o,ex} = \frac{\partial k_{o,ez}(k_{o,ex}, k_{yd})}{\partial k_{o,ex}} \bigg|_{k_{o,ex}=0}, \ a_{o,ey} = \frac{\partial k_{o,ez}(0, k_{o,ey} + k_{yd})}{\partial k_{o,ey}} \bigg|_{k_{o,ey} = k_{yd}}.$$
(31)

The optical field of the photon pairs outside of the optical crystal in the case of a Gaussian pump beam is given by

$$\vec{E}^{(+)}(\vec{r}) \propto \vec{\mathcal{E}}_s(\vec{r}) + \vec{\mathcal{E}}_i(\vec{r}), \tag{32}$$

where

$$\vec{\mathcal{E}}_{s}(\vec{r}) = \int dk_{x} \int dk_{y} \left[b_{o}(k_{x}, k_{y} + k_{yd}) \left(\vec{e}_{ys} + \vec{e}_{x} \frac{(k_{yd} + k_{y} - k_{x}) \cot \theta}{n_{o}k_{0}} \right) + b_{e}(k_{x}, k_{y} + k_{yd}) \right] \times \left(\vec{e}_{x} - \vec{e}_{ys} \frac{(k_{yd} + k_{y} - k_{x}) \cot \theta}{n_{o}k_{0}} \right) \exp(i\vec{k}_{s} \cdot \vec{r}) \exp[i(k_{x}x + k_{y}y)] \tag{33}$$

and

$$\vec{\mathcal{E}}_{i}(\vec{r}) = \int dk_{x} \int dk_{y} \left[b_{o}(k_{x}, k_{y} + k_{yd}) \left(\vec{e}_{yi} - \vec{e}_{x} \frac{(k_{yd} + k_{y} - k_{x}) \cot \theta}{n_{o}k_{0}} \right) + b_{e}(k_{x}, k_{y} + k_{yd}) \right] \times \left(\vec{e}_{x} + \vec{e}_{yi} \frac{(k_{yd} + k_{y} - k_{x}) \cot \theta}{n_{o}k_{0}} \right) \exp(i\vec{k}_{i} \cdot \vec{r} + \phi) \exp[i(k_{x}x + k_{y}y)] \tag{34}$$

V. POLARIZATION CORRELATION

Having the expression for the quantum state and optical field outside the optical crystal for photon pairs generated from SPDC, we can analyze now the polarization correlation between the signal and idler beams. For simplicity, we consider the case of a plan wave pump beam.

According to quantum transition theory, the probability $P(\alpha, \beta)$ of finding simultaneously a photon in the signal beam (the optical beam the wave vector \vec{k}_s) with a polarization in

the direction $\vec{e}_s = \vec{e}_x \cos \alpha + \vec{e}_{ys} \sin \alpha$ and a photon in the idler beam (the optical beam the wave vector \vec{k}_i) with a polarization in the direction $\vec{e}_i = \vec{e}_x \cos \beta + \vec{e}_{yi} \sin \beta$ is proportional to

$$\langle \psi | [\vec{e}_i \cdot \vec{\mathcal{E}}_i^{\dagger}(\vec{r})] [\vec{e}_s \cdot \vec{\mathcal{E}}_s^{\dagger}(\vec{r})] [\vec{e}_s \cdot \vec{\mathcal{E}}_s(\vec{r})] [\vec{e}_i \cdot \vec{\mathcal{E}}_i(\vec{r})] | \psi \rangle. \tag{35}$$

By applying relations (23),(25)and(26) we obtain

$$P(\alpha, \beta) \propto \left| \left(\cos \alpha + \frac{k_{yd} \cot \theta}{n_o k_0} \sin \alpha \right) \left(\sin \beta + \frac{k_{yd} \cot \theta}{n_o k_0} \cos \beta \right) \right.$$

$$\left. + \left(\cos \beta - \frac{k_{yd} \cot \theta}{n_o k_0} \sin \beta \right) \left(\sin \alpha - \frac{k_{yd} \cot \theta}{n_o k_0} \cos \alpha \right) \right|^2$$

$$= \left[1 + \left(\frac{k_{yd} \cot \theta}{n_o k_0} \right)^2 \right] (\cos \alpha \sin \beta + \cos \beta \sin \alpha)^2$$

$$\propto \sin^2(\alpha + \beta). \tag{36}$$

One may observe that by inserting a suitable wave plate into the signal (or idler) beam to swap the x- and y-component of the optical field, or to introduce a phase difference equal to π between these two components, correlations like $\sin^2(\alpha - \beta)$, $\cos^2(\alpha + \beta)$, $\cos^2(\alpha - \beta)$ can also be obtained. All these correlations have been observed in experiments [5], but they are not real evidences for quantum non-locality, because those two photons generated from SPDC are not spatially separated, and both of them can be found in the same signal or idler beam at the same time. The probability P_s of finding both photons in the signal can be calculated by using relations (23) and (25), we have

$$P_{s}(\alpha, \alpha') \propto \langle \psi | [\vec{e}'_{s} \cdot \vec{\mathcal{E}}_{s}^{\dagger}(\vec{r})] [\vec{e}_{s} \cdot \vec{\mathcal{E}}_{s}^{\dagger}(\vec{r})] [\vec{e}'_{s} \cdot \vec{\mathcal{E}}_{s}(\vec{r})] [\vec{e}'_{s} \cdot \vec{\mathcal{E}}_{s}(\vec{r})] | \psi \rangle$$

$$\propto \left| \left(\cos \alpha + \frac{k_{yd} \cot \theta}{n_{o} k_{0}} \sin \alpha \right) \left(\sin \alpha' - \frac{k_{yd} \cot \theta}{n_{o} k_{0}} \cos \alpha' \right) \right.$$

$$\left. + \left(\cos \alpha' + \frac{k_{yd} \cot \theta}{n_{o} k_{0}} \sin \alpha' \right) \left(\sin \alpha - \frac{k_{yd} \cot \theta}{n_{o} k_{0}} \cos \alpha \right) \right|^{2}$$

$$\propto \sin^{2}(\alpha + \alpha' - \gamma), \tag{37}$$

where $\vec{e}_s = \vec{e}_x \cos \alpha + \vec{e}_{ys} \sin \alpha$ and $\vec{e}_s' = \vec{e}_x \cos \alpha' + \vec{e}_{ys} \sin \alpha'$ are two vectors of unity in the polarization directions of two photons, and

$$\gamma = \tan^{-1} \frac{2k_{yd}}{\sqrt{n_o^2 k_0^2 \tan^2 \theta - k_{yd}^2}}.$$
 (38)

VI. CONCLUSION

We presented a quantum theory for the quantum state of photon pairs generated from SPDC, in which the effect of the finite size of optical crystals is explicitly taken into consideration. Explicit expressions for the quantum state and optical fields of these photon pairs are obtained, and we found that these photon pairs are not in entangled quantum states. We analyzed the correlation between the polarizations of the photon in signal beam and the photon in the idler beam, and we showed that this correlation is not an evidence for quantum non-locality as the two photons generated from SPDC are not spatially separated. Results on quantum state of photon pairs generated from SPDC obtained in this paper can also be applied to explain other apparently non-local phenomena, such as "ghost" interference and diffraction [6], four photon entanglement [7, 8] and De Broglie wavelength [9]. We will present those works in separate papers.

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